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FINAL REPORT

APPLICATION OF RISK ANALYSIS IN THE ACQUISITION OF

MAJOR WEAPON SYSTEMS

Prepared by: Dr. George H. Worm
Academic Rank: Associate Professor
Department and: Department of Industrial Management
University: Clemson University
Research Location: AFBPMC/RDCB, Wright-Patterson AFB, OH 45433
USAF Research: David Krahenbuhl, Lt Col; Bill Harris, Myron Bailey
Colleague: -
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APPLICATION OF RISK ANALYSIS IN THE

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by

Dr. George H. Worm

ABSTRACT

An implementation of a statistical approach to cost risk analysis is developed in this paper. A general discussion of risk analysis is presented to familiarize the price analysis with the concepts involved and then forms are presented which allow for the implementation of a risk analysis. Appropriate definitions are given along with a step-by-step procedure. The results of the risk analysis are related to the effect of incentive contracts and several examples are presented.

ACKNOWLEDGEMENTS

I would like to dedicate this work to the memory of Captain William L. Glover, who helped me greatly in the initial phases of this research. His death has affected me greatly and I will never forget him.

I would also like to thank Col Dean Martin, Lt Col David Krahenbuhl, Mr. Bill Harris, Mr. Myron Bailey, and all of the price analyst who gave freely of their time during my investigation. Also I would like to thank the staff of the Business Research Management Center.

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I. INTRODUCTION

During the development and procurement phases of acquiring a major defense weapon system, many decisions must be made concerning its performance, cost and scheduling. An important aspect of this decision making process is the analysis of uncertainty* which exists. Common approaches to analyzing uncertainty have in the past focused on the decision maker's intuition, on sensitivity analysis and on risk analysis. At the time of negotiating one of possibly many contracts, the focus is narrowed to the analysis of uncertainty inherent in the projection of cost involved in a contract. The cost of a specific contract and not the cost of the entire system acquisition is discussed here.

When the term risk analysis is used, three types of risk are generally implied. These types are technical risk, cost risk, and schedule risk. In the study of a large weapon system, all of these risks should be analyzed to determine which alternatives should be chosen in order to maximize the probability of having a successful program. When a specific contract for a program is being negotiated, the primary variable of interest is the cost risk. Important to note, however, is that the cost is not independent of the amount of technical and schedule risk. The technical and schedule risk are important factors in the estimation of the cost risk and hence the cost which should be negotiated.

For a major weapon acquisition both a price and cost analysis are required. These forms of analyses are methods of investigating historical data and projected costs in order to obtain independent estimates of costs from those provided by the contractor.

The cost analysis is an examination of individual cost elements to determine if the estimates approximate the dollars it should cost to perform the contract if the company operates with reasonable economy and efficiency. In the process of a cost analysis there are many uncertainties which may arise and which are not under the control of the government or the contractor. These uncertainties should be isolated in addition to

* Technically many authors differentiate between risk and uncertainty, but the terms will be used interchangeably in this paper.

the cost estimates during the cost analysis. It is important to note that throughout this paper the controllable factors which influence the cost during the performance of a contract are assumed to be at an economic and efficient level on both the part of the government and the contractor.

The price analysis, which is based on comparisons with similar products or earlier production, may provide additional information concerning the estimates and the amount of randomness which might be expected. The price and cost analysis provide the necessary subjective and objective information for a risk analysis. This includes not only the cost estimates but also the definition of random factors which are important influences on cost.

At ASD a price analyst has the responsibility of determining and negotiating a fair and reasonable cost and profit for a contract. Many of the cost elements may be estimated with some degree of certainty and will be referred to as non-random. Examples of non-random cost include negotiated overhead rates, wage rates and certain routine labor costs. Other cost elements will not be known or identifiable with certainty and will be referred to as random. The randomness in the cost elements may be caused by some factor affecting cost or may be totally unexplainable. Causes of randomness in cost elements include design, labor and material uncertainties concerning costs and amounts. The price analyst must still estimate the cost and negotiate a price for the contract. The purpose of this paper is to explain how risk analysis can be used to reflect the extent to which randomness affects total cost.

Random factors affecting cost are events which the contractor cannot control and which are known (or suspected) to impact one or more elements of a cost of the contract. These factors represent the element of risk involved in a contract, thus the term risk analysis.

Again, risk analysis is a procedure for analyzing how randomness affects the total cost. An analyst must identify the random, uncontrollable factors and assess the probability of different events occurring. Then using risk analysis, the distribution of the total cost is obtained. Results of a risk analysis may be useful to a price analyst in several ways. First, it will help to show possible actual costs which might occur and the probability that they will occur. Second, it may help in determining the type of contract to offer. Third, expected cost to the government

and expected profit can be determined. And fourth, actual cost can be bounded or given a range over which it will most likely occur.

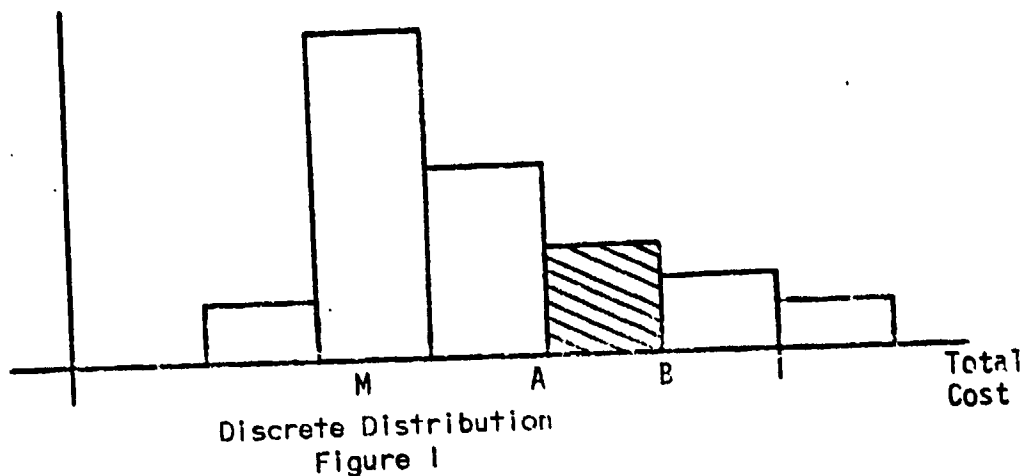
The remainder of this paper will discuss a procedure for risk analysis which avoids the use of simulation by applying some well known statistical properties. First, a structure for risk analysis is discussed which is then applied to a simple case. Second, a cost model is given which allows for a systematic and consistent method of estimating costs and arriving at a total cost. Third, a statistical approach to risk analysis is presented with the accompanying forms for performing the necessary calculations. Finally, examples are given to illustrate the results of a risk analysis.

II. BACKGROUND FOR RISK ANALYSIS

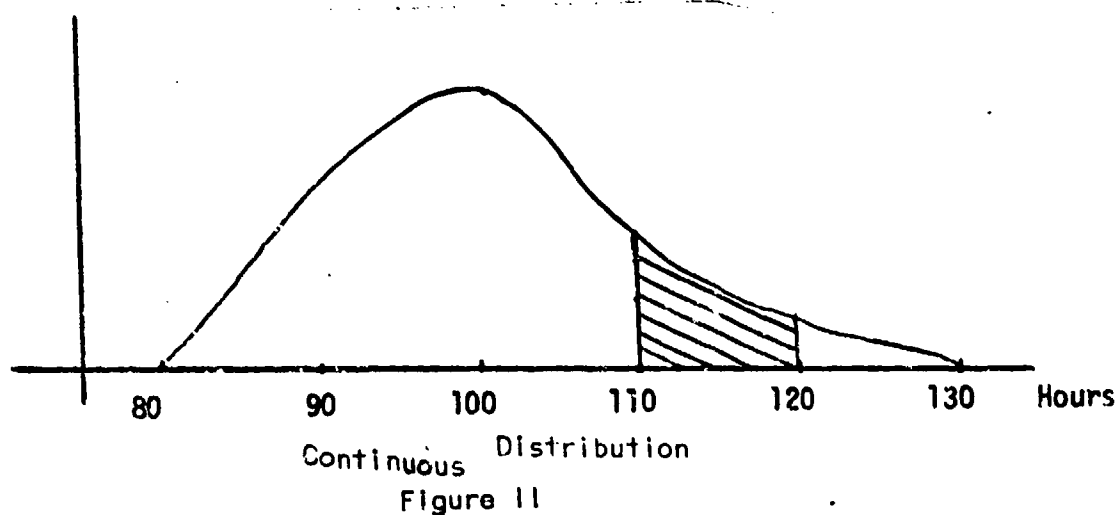
For risk analysis the contract cost must be viewed as an unknown (at the time of cost and/or price analysis) which will be some specific dollar value in the future. That specific dollar value we will assume is the total cost recognized in the final settlement. If this cost was known with certainty and the profit was agreed upon, the price analyst would be out of business. However, since the cost is not known, the price analyst must estimate cost and degree of risk involved.

At the time of negotiation, the future actual cost, since unknown, must be estimated. Risk analysis does not exactly estimate the cost but rather estimates its distribution. A distribution is a pictorial representation of the probabilities of different true costs occurring in the final settlement. This distribution is the ultimate goal of risk analysis.

An example of the output of a risk analysis is shown in the graph in Figure I. This is an example of a discrete distribution, which is simpler than a continuous distribution, and is used when there are discrete outcomes of the random factors. A discussion of a continuous distribution is given later in this section. The height of the curve represents the relative likelihood of occurrence at each cost level. However, the area under the curve gives us probabilities of the cost recognized at the final settlement being between two numbers. In Figure I, the total area under the curve will always be one and the area shaded is the probability that the final actual cost will be between the two dollar figures "A" and "B." The cost denoted by "M" is sometimes referred to as the most likely value.



Appendix I gives a numerical example using discrete events. In the example we have only discussed factors which could have a finite number of outcomes. In most price analysis, however, the randomness is of a continuous type. For instance, the price analysis might estimate that the number of hours required for a contract are going to be between 80 and 130 hours and will most likely be 100. If the distribution of the hours required is continuous then a distribution of the form in Figure II is commonly used. A commonly used distribution for costs is known as a Beta distribution and has several favorable properties to be discussed later.



The analyst is only responsible for choosing the three points. He must choose the minimum, the most likely and the maximum. As with the discrete distribution, the total area is one square unit, and in Figure II the number of squared units in the shaded area is the probability that the hours required in the contract will actually be between 110 and 120.

If Beta distributions are used in specifying possible outcomes for the random factors, then the total cost distribution which is the output of the risk analysis is going to be continuous. The procedure described in step 5 of Appendix I is quite a bit more complex than for the discrete case because there are an infinite number of possibilities.

III. COST MODEL

Common approaches used in risk analysis to handle continuous distributions are simulation and statistical analysis. Although the same basic approach as given in Appendix I is used in simulation, generally a computer is used to manage the calculations involved. Several computer programs for simulation can be found in the literature for performing risk analysis. In order to avoid customizing a risk analysis to a particular contract, a general cost model is given in this section. This cost model is an organization of all the cost subcomponents into a form which can be used in either a simulation study or a statistical analysis.

By applying some well known statistical properties to the cost model below, an alternative to simulation is employed. A general cost model is first described as a starting point for performing a risk analysis for a contract. The total cost is assumed to be comprised of the following subcomponents:

- a) Material (MAT),
- b) Material Overhead (MATOH),
- c) Interdivision Transfer (IT),
- d) Direct Engineering Labor (DEL),
- e) Engineering Overhead (EOH),

- f) Direct Manufacturing Labor (DML),
- g) Manufacturing Overhead (MOH),
- h) Other Costs (OC), and
- i) General and Administrative Expenses (GAE).

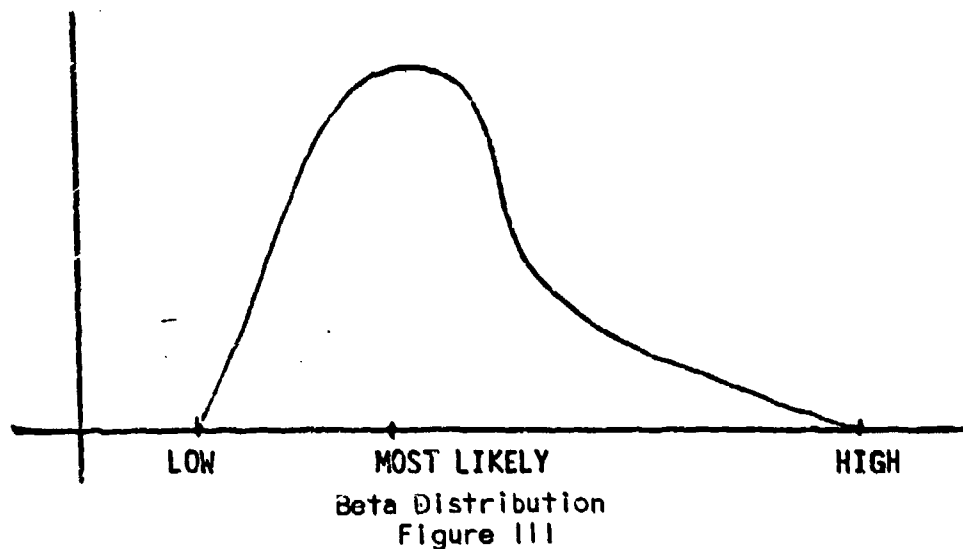
In evaluation of a contract, each of these subcomponents are usually broken down further and are commonly interrelated as shown below, where P1 through P4 are specific percentage figures, and R1 and R2 are specific rates. The * is used to denote multiplication. The general cost model is:

$$\begin{aligned} \text{MAT} &= \text{Estimated Material Cost} \\ \text{MATOH} &= P1 * \text{MAT} + \text{Estimated Independent Material Overhead} \\ \text{IT} &= \text{Estimated IT Cost} \\ \text{DEL} &= (\text{Estimated Engineering Hours}) * R1 \\ \text{EOH} &= P2 * \text{DEL} + \text{Estimated Independent Engineering Overhead} \\ \text{DML} &= (\text{Estimated Manufacturing Hours}) * R2 \\ \text{MOH} &= P3 * \text{DML} + \text{Estimated Independent Manufacturing Overhead} \\ \text{OC} &= \text{Estimated Other Cost} \\ \text{SUBTOTAL} &= \text{ST} = \text{MAT} + \text{MATOH} + \text{IT} + \text{DEL} + \text{EOH} + \text{DML} + \text{MOH} + \text{OC} \\ \text{GAE} &= P4 * \text{ST} \\ \text{TOTAL COST} &= \text{TC} = \text{ST} + \text{GAE} \end{aligned}$$

Even though we are showing the P's and R's as given quantities, they may be considered as random. If they are considered to be random, then a simulation or moments must be used rather than the statistical approach presented here.

Note that in this model eight estimates are needed to determine the total cost. A form for organizing the collection of data required for the risk analysis is given in Form I. The estimates requiring minimum, most likely, and maximum are assumed to be Beta distributed as shown in Figure III. The minimum, most likely, and maximum values must be supplied by the analyst. For each of the cost categories, either the cost (\$) or hours must be estimated. The overhead categories are divided into two parts, the independent overhead cost and the overhead rate. The independent overhead cost is a cost which does not change when the direct cost

changes. Usually, the uncertainty in the independent overhead cost is due to future business conditions. The independent overhead cost is commonly allocated to the direct cost and then lumped with the overhead rate, however, it should be kept separate for a risk analysis. The overhead rate should reflect those costs which are directly proportional to the direct cost. It is assumed here that this rate is known with certainty. Using the Beta distribution implicitly assumes that the possible outcomes can be bounded in some finite range. For mature systems, this is not an unreasonable assumption. Appendix II discusses how these aggregate values can best be estimated.



The formula for the mean (expected) and variance are theoretically based on the properties of the Beta distribution and have been widely used in statistics, risk analysis and scheduling (PERT). The formulas for calculating the Mean and Variance for a Beta distribution are:

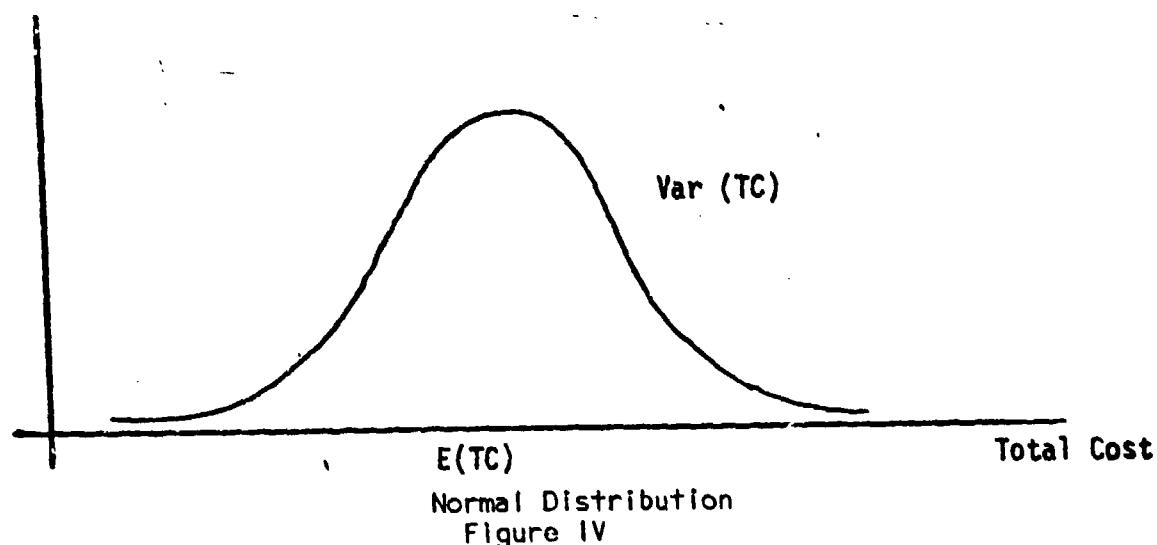
$$\text{Mean} = \frac{L + 4ML + H}{6}$$

and

$$\text{Variance} = \left(\frac{H-L}{6} \right)^2$$

where H=maximum, L=minimum and ML=most likely. These calculations are actually approximations and are very good if the distribution is not too severely skewed ().

The theoretical justification for the use of a statistical method in performing risk analysis is presented in Appendix III. This involves rearranging the cost elements of the total cost so that the total cost is a sum of independent random variables. Using this fact the total cost is known to be normally distributed as shown in Figure IV. This means that only the mean (expected value, $E(TC)$) and variance, $\text{Var}(TC)$ need to be determined in order to make statistical statements about the total actual cost. The next section discusses how the expected value and variance can be estimated from the estimates of the individual cost subcomponents.



IV. Statistical Procedure for Risk Analysis.

As mentioned above, several estimates are required and therefore need to be defined as closely as possible in order to develop good estimates of the actual cost. Figure V shows that there are many different costs involved from the beginning to the end of a contract. They are:

1. Actual,
2. Negotiated,
3. Objective,
4. Most Likely (estimated),
5. Minimum (estimated),
6. Maximum (estimated),
7. Expected (calculated), and
8. Confidence Intervals for Total Cost (calculated).

These eight costs are described below in order to avoid confusion. The costs 4, 5, and 6 are estimates required for a risk analysis and 7 and 8 are calculated in the risk analysis. The information provided by 7 and 8 should be helpful in the establishment of objectives and in the determination of the type of incentive contract to be used.

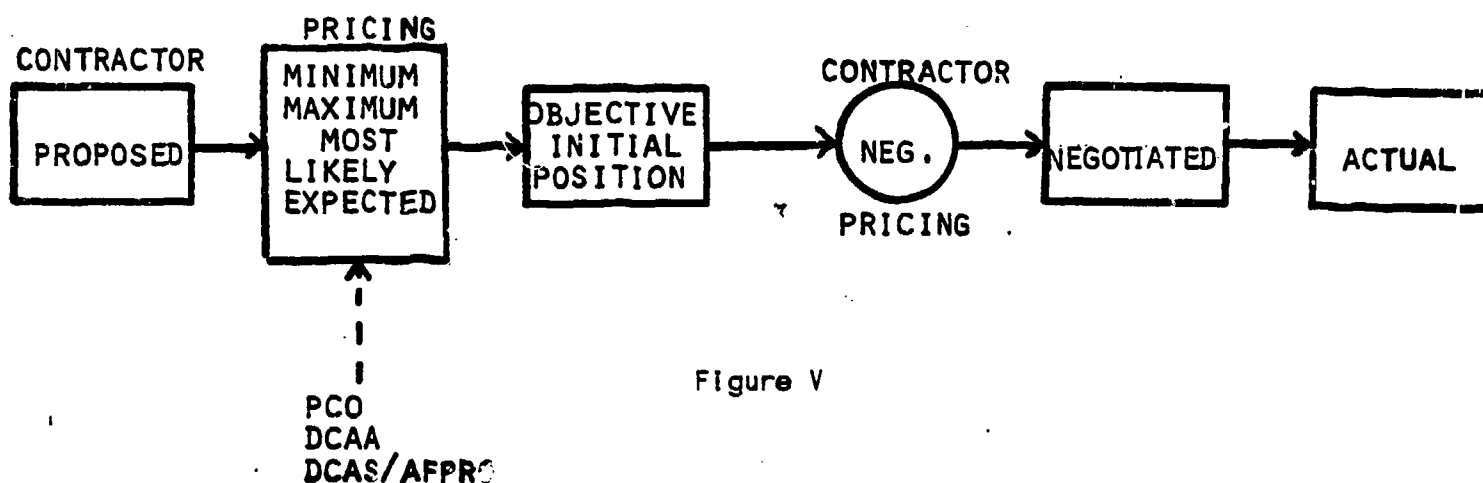


Figure V

1. Actual - The total dollars paid at the time of the final settlement. For incentive contracts the actual is the actual cost plus target profit plus share of underrun or less share of overrun.

2. Negotiated - The negotiated is the target cost, target profit, share and ceiling agreed upon. This will be the basis for the actual settlement as soon as the actual cost is known.

3. Objective - An Air Force goal established before negotiations as an acceptable final negotiated value. Using the proposal and field reports, a fair and reasonable target cost, target profit, ceiling and share are determined. The estimates should be realistic rather than accurate. Realistic assumes that the contractor is responsible for cost control and the creation of operational efficiencies but not necessarily the costs for which he has no control. Accurate estimates tend to encourage inefficiencies and higher prices because the government assumes all cost responsibility.

4. Most Likely - The most likely cost is that projected cost that a contractor may be expected to incur at the completion of the effort under normal, controllable conditions and represents estimated costs that will most likely occur. It does not include uncontrollable risks such as contingencies in the event a vendor does not deliver as scheduled or the quality required, or abnormal cost impacts due to catastrophic conditions. The most likely is an estimate of the actual cost developed using the contractor's proposal, field reports, etc. This estimate is the single most probable cost which might actually occur. There is a tendency to adjust this estimate based on the less probable costs which might occur, however, this adjustment should not be made in the most likely estimate. The most likely estimate should be the estimate of cost which will most likely be correct. Here we are not interested in getting close but are interested in the cost which has the highest probability of actually occurring. This estimate should assume efficiencies and cost control on the part of the contractor.

Operationally the objective cost can be used as the most likely if the following guidelines are followed:

- A. A predetermined level of efficiency and cost control is assumed for both the government and contractor.
- B. The cost estimate is based on expected conditions under which the contractor will have to operate.
- C. No adjustments are made to the most likely cost based on what "might" happen during the performance of the contract.

Note that for negotiations the most likely may or may not be the objective depending on the information obtained from the risk analysis.

5. Minimum - The minimum estimate of the actual cost should be the cost expected under the "best" possible conditions during the performance of the contract. The same level of efficiency and cost control as used for the most likely estimate should be assumed for the contractor's behavior. The reasons that the actual cost would be at the minimum are not controllable by the contractor and will be this low only because of chance. Be sure not to confuse this minimum with the least possible negotiated cost. - Again, the estimate has nothing to do with what might happen during the negotiations but rather should reflect the least possible actual cost which might occur.

Operationally the minimum can be estimated by:

- A. Assuming efficiency and cost control by the contractor during the performance of the contract. (i.e., What is fair and reasonable behavior on the contractor's part?)
- B. Determining the best possible conditions (uncontrollable) which might exist during the performance of the contract and estimating the minimum cost.

Note that the actual cost might be lower than the minimum if the contractor performs more efficiently than assumed. The minimum defined here is for a given contractor's behavior and is the minimum over uncontrollable conditions.

6. Maximum - The maximum estimate of the actual cost should be the cost expected under the "worst" possible conditions during the performance of the contract. Again, the same level of efficiency and cost control should be assumed for the contractor's behavior. The maximum cost would not be due to poor performance by the contractor but would be high strictly because of chance. The maximum is not to be confused with the limits placed on the negotiator as his maximum position or the maximum approved position.

Operationally the maximum can be estimated by:

- A. Assume efficiency and cost control by the contractor during the performance of the contract (i.e., What is fair and reasonable behavior on the contractor's part?)
- B. Determining the worst possible conditions (uncontrollable) which might exist during the performance of the contract and estimating the maximum cost.

Note that the only reason the cost might be higher than the maximum is if there is poor contractor efficiency or cost control. The maximum defined here is for a given contractor's behavior and is the maximum over uncontrollable conditions.

7. Expected - The expected total cost is the average total cost which would occur if the contract were performed many times. This calculated cost may differ from the sum of the most likely cost because it incorporates the randomness involved in each of the cost subcomponents. Actually, the expected total cost is the sum of the expected cost for each subcomponent. From the estimates defined above in 4, 5, and 6 the expected cost can be calculated using a weighted average of those estimates. The formula which is commonly used weights the maximum and minimum equally and weights the most likely by a weight of four. That is:

$$\mu = \text{Expected Cost} = \frac{H + 4ML + L}{6}$$

No truly intuitive feel for this formula can be given, but by weighting the most likely four times as much as the maximum and minimum, the expected cost is pulled away from the midpoint between the maximum and minimum towards the most likely. This formula has been found to work well when the distribution of the cost is not too skewed. As mentioned earlier, the formula for expected cost is theoretically based on the properties of the Beta distribution and has been widely used in statistics, risk analysis, and scheduling (PERT).

The expected total cost will generally be in the center of the possible total costs which might occur. Note that the most likely is not necessarily in the center but is most probable. The expected cost takes into account possible high and (or) low costs which might occur.

8. Confidence intervals for total cost - Just as the expected cost is a measure of the center of the costs which might occur, the variance is a measure of the amount of dispersion in the cost. The variance of individual cost subcomponents can be estimated by dividing the range between the maximum and minimum by six and squaring the results, that is

$$\sigma^2 = \text{Variance} = \left(\frac{H - L}{6} \right)^2$$

The variance of the total cost is then the sum of the variance of the subcomponents. The expected value and variance then totally describe the total because the total is a sum of independent subcomponent costs. The total cost is therefore normally distributed.

A confidence interval is a range of costs which has an associated probability that the actual cost incurred will be in the range. Since the total cost is normally distributed these probability statements are as given below:

STATEMENT	PROBABILITY
Total Incurred Cost $\leq \mu$.5
Total Incurred Cost $\leq \mu + 1\sigma$.8413
Total Incurred Cost $\leq \mu + 2\sigma$.9772
Total Incurred Cost $\leq \mu + 3\sigma$.9987

For example, of all of the possible outcomes of the cost subcomponents, 84.13% would have a total cost of less than $u + 1\sigma$ or there is a probability of .8413 that the actual total cost incurred will be less than $u + 1\sigma$.

The following is a brief description of Forms I, II, and III which can be used as a risk analysis. The accuracy of probability statements depend on the accuracy of the estimates required. The theoretical considerations of the risk analysis are presented in Appendix III.

The purpose of DD Form 633 is to provide a standard format which the contractor submits to the Government a summary of incurred and estimated costs. Form I attached has the same cost categories but allows for uncertainty in the cost estimates. The only difference is the segmentation of overhead into independent overhead costs and overhead rates. The independent overhead cost does not depend on the direct cost and the overhead rates are the factor applied to the direct cost.

STEP 1: Complete the first three columns of Form I

The estimates required in Form I must be made by the price analyst and may require judgemental factors along with mathematical or other methods of cost estimation. It is assumed that the cost will turn out to be some where between the minimum and maximum bounds estimated. The overhead rates are assumed to be known with certainty.

STEP 2: Calculate last two columns of Form I

The formulas for calculating the mean and variance are supplied at the top of Form I.

STEP 3: Transfer values from Form I to the first two columns of Form II and III

STEP 4: Calculate third column of Forms II and III when multiplied by the factor shown.

The third column is the product of the first two and the totals provide an estimate of the Expected Total Cost, $E(TC)$, and the variance of Total Cost, $Var(TC)$.

The "true total cost" will be normally distributed with mean $E(TC)$ and variance $Var(TC)$. With these estimates of $E(TC)$ and $Var(TC)$ the following probability statements can be made.

STATEMENT	PROBABILITY
True Total Cost $E(TC)$.5000
True Total Cost $E(TC) + \sqrt{Var(TC)}$.8413
True Total Cost $E(TC) + 2\sqrt{Var(TC)}$.9772
True Total Cost $E(TC) + 3\sqrt{Var(TC)}$.9987

The Forms I, II, and III can be used to find the expected total cost and the variance and probability statements can be made as above. In addition this information can be used to evaluate different incentive contracts by applying the formulas presented in Appendix III. Since these formulas are very difficult to use, a computer program is given in Appendix IV for calculating the expected profit and expected price for a given incentive contract.

Before discussing the use of this program the concept of expected profit and expected price need to be discussed. As we saw in the example in section II there are many methods of estimating the profit on a contract (see Methods 1-4). The expected profit weighs each possible profit by the probability of the corresponding cost occurring. The expected profit is an average profit if the contract were performed many times. The expected price is the average cost to the government if the contract were performed many times.

An example run of the program is given in Appendix V, where input supplied by the user is underlined and the response of the computer is not. The output in the example is self-explanatory. The high and low are computed at plus and minus $3\sqrt{Var(TC)}$. In other words we can be 99.7% confident that the profit and price will be between the high and low values.

Appendix VI is a listing of a computer program which can be used to perform the calculations involved in Forms I, II, and III. The output of the program is a suggested price ceiling and contractor share calculated from the risk analysis. Note that the incentive contract is suggested for contracts which have more than a five percent variation. An example run is given in Appendix VII. The inputs required in addition to the entries in Form I are the Weighted Guideline Method (WGM) profit and the cost risk used in the WGM. The program computes a ceiling and contractor's share based on the concept that $E(TC) + 3\sqrt{\text{Var}(TC)}$ is the point of total assumption and the corresponding profit should be the WGM profit less the cost risk.

The next section presents several examples of the use of Forms I, II, and III and discusses the conclusions which can be drawn from each analysis.

V. EXAMPLES

The following examples are actual cases which have been negotiated or are in the process of being negotiated by ASD.Pricing. The specifics concerning the companies and airframes involved are not disclosed here. The estimates in the first three columns of Form I were given by knowledgeable price analysts involved with the negotiations. Cost entries are in \$1,000,000 or hours are in 1,000,000 hours. The wage rates are in dollars.

Case 1

The information obtained from the price analyst for case one is shown in the first three columns of Case 1-I. The material, engineering and manufacturing overheads for this case were considered to be independent. It would be preferable to breakdown the overhead into two separate categories independent and rate applied to a base, however, this information was not available. The effect on the analysis of treating all of the overhead as independent is not extremely significant but will cause the confidence intervals to be tighter than if it were separated.

The hours for labor were not readily available, therefore, the cost of labor was used. This will not have any effect on the results of the analysis.

From the estimates provided, the last two columns of Case 1-I and forms Case 1-II and Case 1-III were completed. The analysis shows that the expected total cost is 38.0266 and the variance is .2877. The following probability statements can then be made.

<u>STATEMENT (\$1,000,000)</u>	<u>PROBABILITY</u>
Total Cost \leq 38.0266	.5000
Total Cost \leq 38.563	.8413
Total Cost \leq 39.1	.9772
Total Cost \leq 39.635	.9987

Observe that there is a very small chance of the cost exceeding the expected by more than 4 percent, i.e. $(3\sqrt{\text{Var}(TC)}/E(TC))$. With this small risk most likely a FFP would be preferred. This information can be used to decide on the type of contract and to evaluate different share ratios and ceilings. For instance the following information was obtained using the computer program in appendix IV. The share for over and under runs was kept the same in this example.

TARGET COST	TARGET PROFIT	CONTRACTOR'S SHARE	CEILING	PROFIT EXPECTED	PRICE EXPECTED
38.5	3.85	.4	46.	4.028	42.054
38.0	3.8	.4	39.635	1.693	39.719
38.5	4.2	.3	42.	3.959	41.986
38.5	3.85	.3	42.	3.891	41.918
38	3.8	.3	39.635	1.6725	39.699
38.5	4.2	.4	42.	3.961	41.989
38.5	4.2	.3	46.	4.329	42.356

Note that the expected profit and the expected price are all approximately the same, indicating that the FFP contract would be preferred as mentioned earlier. Although the second and fifth contracts appear to be less costly, the ceiling would be far to tight (approximately 104%).

ESTIMATES FOR RISK ANALYSIS

ESTIMATES

(\$1,000,000)

$$\text{EXPECTED VALUE} = \frac{L + 4ML + H}{6}$$

$$\text{CALCULATED VARIANCE} = \left(\frac{H - L}{6} \right)^2$$

SUBCOMPONENTS	MINIMUM (L)	MOST LIKELY (ML)	MAXIMUM (H)	EXPECTED VALUE	CALCULATED VARIANCE
MATERIAL	7.9	8.0	8.1	8.0	.0011
MATERIAL OVERHEAD	1.2	1.3	1.4	1.3	.0011
RATE FOR MATERIAL	P1				
INTERDIY TNSFRS	INDEPENDENT				
DIRECT ENGRG LABOR	2.6	3.6	3.6	3.43	.0278
WAGE RATE	R1				
ENGRG OVERHEAD	4.1	5.6	5.6	5.35	.0625
RATE FOR ENGRG	P2				
DIRECT MFG LABOR	3.9	4.6	4.7	4.5	.0178
WAGE RATE	R2				
MFG OVERHEAD	8.9	10.2	10.9	10.1	.1111
RATE FOR MFG	P3				
OTHER COSTS	.66	.68	.68	.6767	.00001
G&A EXPENSES	COST				
PERCENT OF SUBTOTAL	P4				

FORM I
CASE I-I

RISK ANALYSIS WORKSHEET (VARIANCE)

(COL 1)	(COL 2)	(COL 3) = (COL 1) * (COL 2)
$(1 + P1)^2$	Var(V1)	
<u>1</u>	<u>.0011</u>	<u>.0011</u>
	Var(V2)	
<u>1</u>	<u>.0011</u>	<u>.0011</u>
	Var(V3)	
<u>1</u>	<u>.0278</u>	<u>.0278</u>
$(R1 + R1*P2)^2$	Var(V4)	
<u>1</u>	<u>.0625</u>	<u>.0625</u>
	Var(V5)	
<u>1</u>	<u>.0178</u>	<u>.0178</u>
$(R2 + R2*P3)^2$	Var(V6)	
<u>1</u>	<u>.1111</u>	<u>.1111</u>
	Var(V7)	
<u>1</u>	<u>.00001</u>	<u>.00001</u>
	Var(V8)	
	<u>.00001</u>	<u>.00001</u>
	TOTAL (COL 3)	
	$(1 + P4)^2$	
	<u>.22142</u>	<u>.22142</u>
	<u>.11145</u>	<u>.11145</u>
	<u>.2974</u>	<u>.2974</u>

VARIANCE OF TOTAL COST = Var(TC) = $(1 + P4)^2$ * TOTAL (COL 3) = .2974

FORM III

CASE 1-III

RISK ANALYSIS WORKSHEET (EXPECTED VALUE)

(COL 1)	(COL 2)	(COL 3) = (COL 1) * (COL 2)
(1 + P1) <u> </u>	E(V1) <u>8.0</u>	<u>8.0</u>
<u>1</u>	E(V2) <u>1.3</u>	<u>1.3</u>
<u>1</u>	E(V3) <u> </u>	<u> </u>
(R1 + R1*P2) <u> </u>	E(V4) <u>3.43</u>	<u>3.43</u>
<u>1</u>	E(V5) <u>5.35</u>	<u>5.35</u>
(R2 + R2*P3) <u> </u>	E(V6) <u>4.5</u>	<u>4.5</u>
<u>1</u>	E(V7) <u>10.1</u>	<u>10.1</u>
<u>1</u>	E(V8) <u>.6767</u>	<u>.6767</u>
	TOTAL (COL 3)	<u>33.3567</u>
	(1 + P4)	<u>1.14</u>
	EXPECTED TOTAL COST = E(TC) = (1 + P4)* TOTAL (COL 3)	<u>38.03</u>

FORM II

CASE I-II

Case 2

The information obtained from the price analyst for Case 2 is shown in the first three columns of Case 2-I. In this example the overheads were not considered to be independent and fringe benefits were a factor applied to labor. Note that the fringe rate was added to the overhead rate although it was in other costs originally.

From the estimates provided, the total cost using the minimum, most likely, and maximum were 18,579; 19,532; and 20,430 respectively. After completing Case 2-II and Case 2-III the resulting expected value and variance were 19,525 and 33,327. The following probability statements can then be made.

<u>STATEMENT (\$1,000)</u>	<u>PROBABILITY</u>
Total Cost \leq 19,525	.5
Total Cost \leq 19,707	.8413
Total Cost \leq 19,890	.9772
Total Cost \leq 20,072	.9987

Note that the total cost obtained from the maximum positions 20,430 would be extremely improbable if the contractor is efficient and uses cost controls.

This case has very little uncertainty because the extreme case of 20,072 is only 2.8% ($3 \sqrt{\text{Var}(\text{TC})}/E(\text{TC})$) larger than the expected cost. This would indicate a FFP type of contract would most likely be acceptable. The maximum gain or loss for the contractor for a FPIF contract would be his share of 2.8% of the expected cost. For instance, if the share ratio were 40% the contractor may gain or lose atmost (atmost means with probability less than .13%) 1.12% profit due to the uncertainty in this contract.

ESTIMATES FOR RISK ANALYSIS

(\$1,000)

ESTIMATES

$$\text{EXPECTED VALUE} = \frac{L + 4ML + H}{6}$$

$$\text{CALCULATED VARIANCE} = \left(\frac{H - L}{6} \right)^2$$

MOST

MINIMUM (L) LIKELY (ML) MAXIMUM (H)

SUBCOMPONENTS

MATERIAL COST

MATERIAL OVERHEAD INDEPENDENT

RATE FOR MATERIAL 11.03 P1

INTERDIV INSFRS

DIRECT ENGRG LABOR

WAGE RATE

ENGRG OVERHEAD

RATE FOR ENGRG

DIRECT MFG LABOR

WAGE RATE

MFG OVERHEAD

RATE FOR MFG

OTHER COSTS

G&A EXPENSES

PERCENT OF SUBTOTAL

11.03 P4

8,367

8,867

9,267

8,850.3

22,500

1751

1905

2060

1,905.2

2,652.3

34

39

45

39.16

3.4

11.57 R1

INDEPENDENT

11.03 P2

spring .4

214

217

221

217.2

1.4

11.03 R2

INDEPENDENT

11.03 P3

spring .4

420

450

480

450

100

FORM I

CASE 2-I

23

26

RISK ANALYSIS WORKSHEET (EXPECTED VALUE)

(COL 1)	(COL 2)	(COL 3) = (COL 1) * (COL 2)
(1 + P1) <u>1.045</u>	E(V1) <u>8,850.3</u>	<u>9,248.3</u>
<u>1</u>	E(V2) <u> </u>	<u> </u>
(R1 + R1*P2) <u>23.49</u>	E(V3) <u>1,905.2</u>	<u>1,905.2</u>
<u>1</u>	E(V4) <u>39.16</u>	<u>919.9</u>
(R2 + R2*P3) <u>28.02</u>	E(V5) <u> </u>	<u> </u>
<u>1</u>	E(V6) <u>217.2</u>	<u>6,085.9</u>
<u>1</u>	E(V7) <u> </u>	<u> </u>
<u>1</u>	E(V8) <u>450</u>	<u>450</u>
	TOTAL (COL 3)	<u>18,609.6</u>
	(1 + P4)	<u>1.0492</u>
	EXPECTED TOTAL COST = E(TC) = (1 + P4) * TOTAL (COL 3)	<u>19,545</u>

FORM II

CASE 2-II

RISK ANALYSIS WORKSHEET (VARIANCE)

(COL 1)	(COL 2)	(COL 3) = (COL 1) * (COL 2)
$(1 + P1)^2$ <u>1.092</u>	Var(V1) <u>22500</u>	<u>24,570</u>
<u>1</u>	Var(V2) <u> </u>	<u> </u>
$(R1 + R1*P2)^2$ <u>551.78</u>	Var(V3) <u>2,652.3</u>	<u>2,652.3</u>
<u>1</u>	Var(V4) <u>3.4</u>	<u>1,676.</u>
$(R2 + R2*P3)^2$ <u>784.9</u>	Var(V5) <u> </u>	<u> </u>
<u>1</u>	Var(V6) <u>1.4</u>	<u>1098.8</u>
<u>1</u>	Var(V7) <u> </u>	<u> </u>
<u>1</u>	Var(V8) <u>100</u>	<u>100</u>
	TOTAL (COL 3) <u>30,297</u>	
	$(1 + P4)^2$ <u>1.1</u>	
	VARIANCE OF TOTAL COST = Var(TC) = $(1 + P4)^2$ * TOTAL (COL 3)	<u>33,327</u>

FORM III

CASE 2-III

Case 3

The information obtained from the price analysts for Case 3 is shown in the first three columns of Case 3-I. The overhead in this example could not be separated from the costs associated with material, engineering labor, and manufacturing labor. This is no problem in the analysis except for the difference which would occur if the independent overhead were kept separate.

The G&A is not a percent of the subtotal in this example and is treated differently. This example was used to illustrate that to handle special cases the forms may need to be adjusted.

The minimum most likely and maximum positions result in a total cost of 63.2, 66.1, and 71.55 (million) respectively. After completing Case 3-II and Case 3-III the resulting expected value and variance were 66.5 and .6013. The following probability statements can then be made.

<u>STATEMENT (\$1,000,000)</u>	<u>PROBABILITY</u>
True Cost \leq 66.5	.5
True Cost \leq 67.3	.8413
True Cost \leq 68.1	.9772
True Cost \leq 68.9	.9987

Note that the most likely position is .4 million less than the expected cost. This would indicate that the most likely position is a little too low. Again in this example there is very little uncertainty. There is almost (probability less than .0013) no chance that the cost will vary more than 3.6% ($3\sqrt{\text{Var}(TC)}/E(TC)$) because of the uncertainty. It is of interest to note that the maximum position is 3.4 million more than 68.1 which will be exceeded with probability $1 - .9772 = .0228$.

ESTIMATES FOR RISK ANALYSIS

(#1,000,000)

ESTIMATES

SUBCOMPONENTS	MOST			EXPECTED VALUE $\frac{L + 4ML + H}{6}$	CALCULATED VARIANCE $\left(\frac{H - L}{6}\right)^2$	
	MINIMUM (L)	LIKELY (ML)	MAXIMUM (H)		E(V1)	Var(V1)
MATERIAL	39	40.2	42.1	40.3167		.2669
MATERIAL OVERHEAD					E(V2)	Var(V2)
RATE FOR MATERIAL	P1					
INTERDIY TNSRS					E(V3)	Var(V3)
DIRECT ENGRG LABOR	1.5	1.6	1.95	1.6417	E(V4)	Var(V4)
WAGE RATE	R1					
ENGRG OVERHEAD					E(V5)	Var(V5)
RATE FOR ENGRG	P2					
DIRECT MFG LABOR	13.9	15.0	17.1	15.1667	E(V6)	Var(V6)
WAGE RATE	R2					
MFG OVERHEAD					E(V7)	Var(V7)
RATE FOR MFG	P3					
OTHER COSTS	5.3	5.7	6.5	5.7667	E(V8)	Var(V8)
G&A EXPENSES	3.5	3.6	3.9	3.633		.0044
PERCENT OF SUBTOTAL	P4					

FORM I
CASE 3-I

RISK ANALYSIS WORKSHEET (EXPECTED VALUE)

	(COL 1)	(COL 2)	(COL 3) = (COL 1) * (COL 2)
(1 + P1)	<u>1</u>	E(V1) <u>40.3167</u>	<u>40.3167</u>
	<u>1</u>	E(V2) <u> </u>	<u> </u>
	<u>1</u>	E(V3) <u> </u>	<u> </u>
(R1 + R1*P2)	<u>1</u>	E(V4) <u>1.6417</u>	<u>1.6417</u>
	<u>1</u>	E(V5) <u> </u>	<u> </u>
(R2 + R2*P3)	<u>1</u>	E(V6) <u>15.1667</u>	<u>15.1667</u>
	<u>1</u>	E(V7) <u> </u>	<u> </u>
	<u>1</u>	E(V8) <u>5.7667</u>	<u>5.7667</u>
G+H	<u>1</u>	3.633	<u>3.633</u>
		TOTAL (COL 3)	<u>66.5251</u>
		(1 + P4)	<u>1</u>
		EXPECTED TOTAL COST = E(TC) = (1 + P4) * TOTAL (COL 3)	<u>66.5251</u>

FORM II

CASE 3-II

RISK ANALYSIS WORKSHEET (VARIANCE)

(COL 1)	(COL 2)	(COL 3) = (COL 1) * (COL 2)
$(1 + P1)^2$	<u>1</u>	<u>.2669</u>
	<u>1</u>	<u> </u>
	<u>1</u>	<u> </u>
$(R1 + R1*P2)^2$	<u>1</u>	<u>.0056</u>
	<u>1</u>	<u> </u>
$(R2 + R2*P3)^2$	<u>1</u>	<u>.2844</u>
	<u>1</u>	<u> </u>
	<u>1</u>	<u>.04</u>
	<u>1</u>	<u>.0044</u>
GVA	<u>1</u>	<u>.6013</u>
		TOTAL (COL 3)
		<u>1</u>
		$(1 + P4)^2$
		TOTAL (COL 3) * TOTAL (COL 3)
		<u>.6013</u>

VARIANCE OF TOTAL COST = Var(TC) = $(1 + P4)^2$ * TOTAL (COL 3)

FORM III

CASE 3-III

VI. SUMMARY

In this paper an attempt has been made to provide and explain the necessary concepts for a risk analysis. Although the methodology is theoretically sound, the user should be aware of the following pitfalls in attempting to use the forms supplied here.

1. Preconceived definitions of terms used in this paper should be avoided. For the purpose of risk analysis the user should not already know the meaning of the following terms unless he has a strong background in statistics. The technical terms used here include:

- A. Risk analysis,
- B. Probability distribution,
- C. Maximum and minimum costs,
- D. Most likely costs,
- E. Expected cost,
- F. Variance,
- G. Actual total cost as a random variable, and
- H. Independent cost subcomponents.

2. Bias positions in estimates of costs will result in a bias risk analysis.

3. Contractor efficiency and cost control are not an issue in this analysis.

4. As more information becomes available the maximum, most likely, and minimum should be adjusted appropriately and the risk analysis repeated. Additional information will tend to reduce the amount of variability.

5. The risk factor used in the profit weighted guidelines should not be added in addition to the cost which is added because of a risk analysis.

6. This analysis should not be used if the total cost is dominated by one cost subcomponent. Good business sense should always be applied in defining objectives and risk analysis should be recognized as only an information tool for capturing the uncertainty in cost estimations. This tool helps the analyst to see what might happen in order to choose appropriate objects and/or contract types.

Risk analysis can be a very useful tool but the results should always be interpreted with common sense.

Suggested Readings in Cost Risk Analysis

1. Dienemann, P. F., "Estimating Cost Uncertainty Using Monte Carlo Techniques," RM-4854-PR, The Rand Corporation. January, 1966.
2. Fisher, G. H., Cost Considerations in Systems Analysis, American Elsevier Publishing Company, New York.
3. Klein, Michael R., Treating Uncertainty in Cost Analysis: The Beta Distribution, U.S. Navy CNO Resource Analysis Group.
4. Murphy, E. L., Jr., "Statistical Methods of Measuring the Uncertainty of Cost Estimates," AD 718862, February, 1970.
5. Quade, E. S. and Boucher, W. I., System Analysis and Policy Planning--Application in Defense, American Elsevier Publishing Company, New York.
6. Schaefer, D. F., Husic, F. J. and Gutowski, M. F., "A Monte Carlo Simulation Approach to Cost--Uncertainty Analysis," AD 850054, March 1969.
7. Wilder, J. J., An Analytical Method of Cost Risk Analysis, Grumman Aerospace Corporation, PDR-OP-T77-12, June 1977.

APPENDIX I

Discrete Risk Analysis

In discussion of risk analysis, we are concerned with those factors which are random (i.e., which cannot be controlled) and which affect cost in some way. For instance, we are not sure how many labor hours will be required or how much scrap there will be. Once the randomness is determined, a risk analysis can be performed in the following way.

STEP 1 - Identify those random factors which would affect the cost of the contract. (A factor might be the number of hours required by the contract).

These factors should be as independent as possible. That is, the future outcome or value of one factor should not influence the outcome of another. If this is not the case then the dependence of one factor on another must be defined.

STEP 2 - Determine the specific outcomes which might occur for each factor and the probability of each occurring. (The outcomes of hours required could be "high" with probability .3 or "low" with probability .7).

STEP 3 - Break the total cost into subcomponents and define how these subcomponents are interrelated.

STEP 4 - Define which cost subcomponents would be affected by each factor and the magnitude of the effects for each possible outcome.

STEP 5 - The distribution of the total cost (e.g., Figure 1) may be obtained by considering all possible combinations of the states of the factors, calculating the total cost and weighing these costs by the product of the probabilities of the specific outcomes occurring.

Using the above procedures one should make sure that "bias" and randomness are separated. If bias is included in a risk analysis, then it

should be recognized as such and not incorporated as part of the randomness. Risk analysis of a bias position can be performed, however, the end result will also be biased.

A simple example may help to explain exactly what is meant by the steps described in Section II for a discrete distribution. This example is oversimplified in order to demonstrate how a risk analysis could be performed. For the contract under consideration we have three cost sub-components which are interrelated as follows:

1. Labor = hours times wage rate
- Step 2. Materials
- 3
3. Overhead = Labor overhead plus General and Administrative (G&A)

Total cost is then Labor plus Materials plus Overhead. Note that a change in labor hours or wage rate affect Labor and also Overhead.

Suppose that for the contract under consideration, we expect the following:

Labor hours =	100
wage rate =	\$10/hr
Labor Overhead =	150% Labor
materials =	\$5000
G&A =	\$1000

Therefore the cost model above gives us:

Estimated Total Cost = \$8,500

The factors which we have determined to affect the cost of this project are:

1. Uncertain estimate of hours
- Step 2. Negotiated new wage rate with union
- 1 3. Reliability of supplier of materials

The outcomes of these factors are given below with their associated probabilities. Generally these probabilities are subjective estimates by the price analyst.

1. Uncertain estimate of hours

- a) 25% chance hours will be 80
- b) 50% chance hours will be 100 (expected)
- c) 25% chance hours will be 130

STEP
2

2. Negotiated wage rate with union

- a) 70% chance low wage rate (\$10) (expected)
- b) 30% chance high wage rate will occur (\$11)

3. Reliability of supplier of materials

- a) 75% chance supplier will deliver (\$5,000)(expected)
- b) 25% chance the materials will have to be purchased on open market (\$6,000)

Note that the total of the probabilities under each factor total 100%. This must always be the case. The factors above will effect the cost in the following manner.

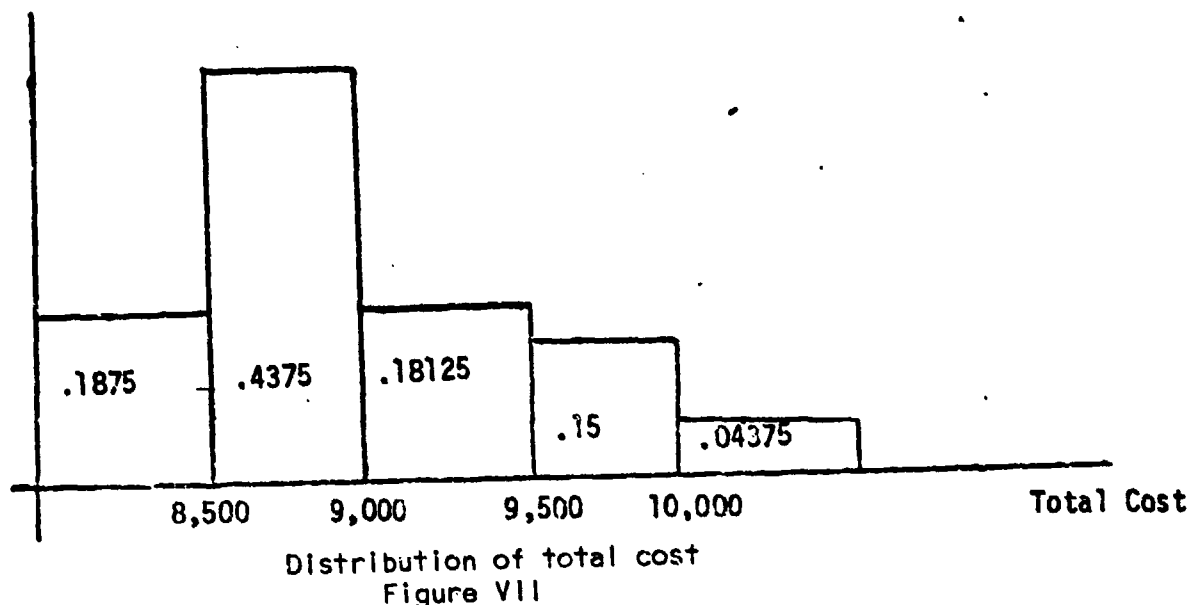
Factor 1 - the outcome of factor number 1 clearly is going to effect the total cost by the amount caused by changing the hours in the cost model defined in step 3.

Step
4

Factor 2 - Suppose that the low wage rate is \$10/hr, but may have to be increased to \$11/hr. The effect on total cost can be determined by changing the wage rate in model given in step 3.

Factor 3 - The current supplier will supply the material at \$5,000, however, if he fails to do so then the materials will cost us \$6,000. This will effect the material cost only.

The possible costs for this contract are given in Table 1 below with their associated probabilities. The resulting probability distribution for total cost of the project is shown in Figure VII. From this graph we can now make statements about the probability that the true final cost will be in different ranges. For instance, the probability that the total cost will be between \$9,000 and \$9,500 is .18125. Also, we can see that there is only a .04375 probability that the total cost will exceed \$10,000. The most likely cost will be between \$8,500 and \$9,000.



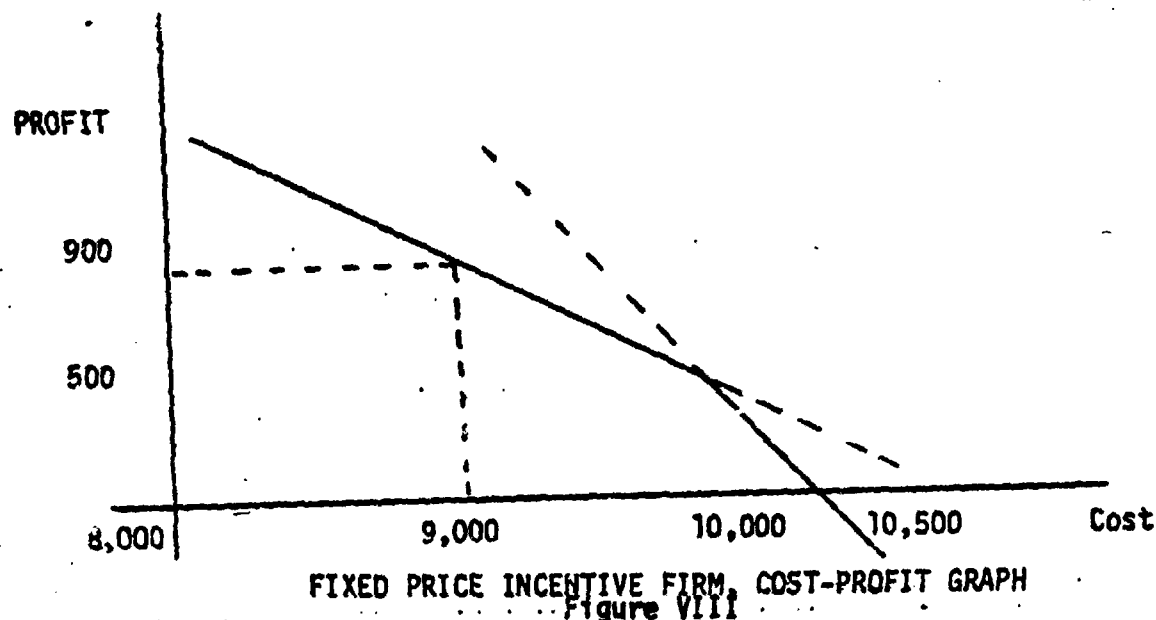
Expected cost is the cost which would occur on the average if this contract were executed many times. It is determined by weighing the total costs in Table I by the associated probability. In this case, the expected total cost is $.05625(8000) + .1125(8500) + \dots = \$8,866.87$.

If we compare the information obtained above and in Figure VII to the total cost using only what we expected, it is clear that our original estimate of the cost was too low (\$8,500).

Let us now evaluate a Fixed Price Incentive Firm contract where

Target Cost = \$9,000,
 Target Profit = \$ 900,
 Price Ceiling = \$10,250, and
 Contractor Share = 30%.

See Figure VIII for the cost/profit graph.



Several possible ways exist for determining the profit and the cost to the government which might be expected.

Method 1 - If we use our original estimates of the cost components the profit appears to be $900 + .3 (9,000 - 8,500) = 1,050$. The corresponding cost to the government would be $8,500 + 1,050 = 9,550$.

Method 2 - If we use the expected cost, the profit appears to be $900 + .3 (9,000 - 8,867) = 940$. The corresponding cost to the government would be $8,867 + 940 = 9,807$.

Method 3 - Using the same method as used to determine the expected cost, the expected profit can be determined by weighing each of the profits determined by the outcomes in Table I by the associated

probabilities. In this case, expected profit would be
 $.05625 (900 + .3(9000-8000)) + .1125 (900 + .3(9000-8500)) +$
 $. . . = 996$ and the expected cost to the government would be
 $8,867 + 996 = \$9,863.$

Method 4 - The best and worst possibilities and the probability of them occurring are given below.

	BEST	WORST
Probability	.05625	.04375
Cost	8,000	10,075
Profit	1,200	175
Cost to Gov't	9,200	10,250

The four methods above are used to show that different estimated costs to the government and different profits can be obtained. Method 1 is a common method which does not include the risk. Methods 3 and 4 are the method of determining the expected profit and cost to the government using risk analysis. In fact, if all of the randomness is contained in the three factors used and it were possible to perform this contract many times, the average profit per contract and the average cost to the government would be the expected values given in Method 3.

APPENDIX II

Aggregated Estimates

Usually, the values required in Form I are not aggregated to the level required. It is necessary therefore to discuss how these aggregate estimates can be determined from much more detailed estimates. The procedures for aggregation are of two types. First is the estimation of maximum, most likely and minimums for a total, where the maximums, most likelies and minimums are estimated for each element of a total. Second is the estimate of aggregate rates or percentages.

For the first type, it would not be correct to estimate the maximum, most likely and minimum by simply adding the corresponding values from each element. For instance, Material may be composed of three elements such as

	Minimum(L)	Most Likely(ML)	Maximum(H)
Subcontracted Items	5,000	6,000	8,000
Purchased Parts	3,000	3,000	3,000
Other Material	10,000	12,000	16,000

Then rather than using the three corresponding totals 18,000, 21,000 and 27,000 and coming up with a mean of 21,500 and a variance of 2,250,000, we would find the mean and variance of each element and use the total, which would be a mean of 21,500 and a variance of 1,250,000. Remember that the mean and variance are calculated as

$$\text{Mean} = \frac{L + 4*ML + H}{6} \quad \text{and}$$

$$\text{Variance} = \left(\frac{H - L}{6} \right)^2$$

In mathematical form the mean and variance for a total of several elements is given by:

$$\text{Mean} = \sum_{i=1}^n \frac{L_i + 4*ML_i + H_i}{6}$$

$$\text{Variance} = \sum_{i=1}^n \left(\frac{H_i - L_i}{6} \right)^2 ,$$

where i denotes the i^{th} element and n is the number of elements of the total. For this type of situation, the maximum, most likely and minimum are not needed since the mean and variance are already determined.

For the second type of aggregation, a weighted average could be used as the single rate or percentage needed. For instance, suppose that the labor hours are broken into two parts

	Hours(H)	Rate(R)
Skilled	300	\$11.00
Non-Skilled	500	\$ 8.00

Then the weighted average rate would be $(300*11 + 500*8)/(300 + 500)$ or 9.125. Note that when this rate is applied to the total hours we get the same as if the two rates were applied separately and then totaled. Mathematically this could be expressed as

$$\text{Average Rate} = \frac{\sum_{i=1}^n H_i * R_i}{\sum_{i=1}^n H_i} ,$$

where i denotes the i^{th} element and n is the number of elements. A weighted average percentage can be obtained in the same fashion.

APPENDIX III

STATISTICAL APPROACH

The approach used in this paper to perform risk analysis is the application of some well known statistical analysis to the cost model described in Section III. The statistical analysis approach is useful in the handling of continuous distribution problems.

The statistical concept used in this paper is one which states that the sum of independent random variables will be approximately normally distributed with a mean equal to the sum of the individual means and variance equal to the sum of the individual variances. Usually for more than four independent random variables this approximation is also very good.

If we rewrite our total cost model, we can get it into the form need. That is:

$$TC = \{(1+P1)*V1 + V2 + V4*(R1 + R1*P2) + V6*(R2 + R2*P3) + V5 + V3 + V7 + V8\} *(1 + P4)$$

where V1 = Material Cost
V2 = Independent Material Overhead
V3 = IT Cost
V4 = Engineering Hours
V5 = Independent Engineering Overhead
V6 = Manufacturing Hours
V7 = Independent Manufacturing Overhead
V8 = Other Costs

Using the notation $E(V1)$ to represent the mean of variable $V1$ and $Var(V1)$ to denote the variance of $V1$, the mean $E(TC)$, and variance, $Var(TC)$, of the total cost are:

$$E(TC) = \{(1+P1)*E(V1) + E(V2) + E(V4)*(R1+R1*P2) + E(V6)* \\ (R2 + R2*P3) + E(V5) + E(V3) + E(V7) + E(V8)\} * (1+P4)$$

$$Var(TC) = \{(1+P1)^2*V(V1) + Var(V2) + Var(V4)*(R1+R1*P2)^2 + Var(V6)* \\ (R2+R2*P3)^2 + Var(V5) + Var(V3) + Var(V7) + Var(V8)\} *(1 + P4)^2$$

The distribution of the total cost would thus be as given in Figure VI.

The normal distribution has the property that probability statements can be made using only the mean and variance. For instance, there is a 68% chance that the total cost will be between $E(TC) \pm \sqrt{Var(TC)}$ and a 95% chance that the total cost will be between $E(TC) \pm 2*\sqrt{Var(TC)}$.

In order to facilitate the calculation of $E(TC)$ and $Var(TC)$, two work sheets are provided in Forms II and III. The values needed for these worksheets can be taken from the form in Form I.

Once the mean and variance of the total cost have been determined, the distribution of the total cost is completely described since the total cost is normally distributed. The functional form of the normal distribution is:

$$f(c) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(c - \mu)^2}{2\sigma^2}}$$

Where $\mu = E(TC)$ and $\sigma^2 = Var(TC)$. This distribution is then the completion of the risk analysis.

Let us now investigate the effect of risk on a Fixed Price Incentive Firm contract which is commonly used for contracts which have some risk but not enough to resort to a cost plus type contract. In particular, the expected profit and the expected cost to the government will be expressed in terms of the following variables:

TC = Target Cost

TP = Target Profit

PC = Ceiling Price

a = Contractor's share of underruns

b = Contractor's share of overruns

CC = Point of total assumption = $\frac{PC - (TC + TP)}{1-b} + TC$

and

f(c) = distribution of true total cost

A profit-cost graph for a Fixed Price Incentive Firm contract is presented in Figure VI. The line segments to the left of TC, between TC and CC and to the right of CC are given in terms of the cost (C) as

$$L1 = TP + a(TC - C)$$

$$L2 = TP - b(C - TC) \text{ and}$$

$$L3 = PC - C.$$

Thus, the expected profit is given by:

$$E(\text{Profit}) = \int_{TC-3\sigma}^{TC} L1 f(c) dc + \int_{TC}^{CC} L2 f(c) dc + \int_{CC}^{TC+3\sigma} L3 f(c) dc$$

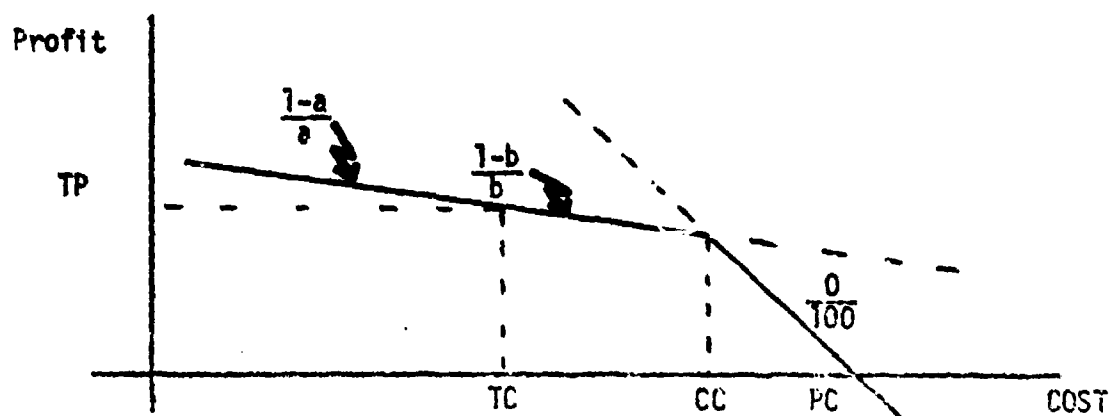
Of course if CC is greater than TC + 3σ, then the last term is dropped.

The expected cost to the government is then

$$E(TC) + E(\text{Profit}).$$

Note that the Normal distribution has been truncated at plus or minus 3σ since it is extremely unlikely that the cost will be outside of that range.

A computer program is provided for the calculations necessary for comparing different incentive plans. This program accepts as input TC, TP, σ, a, b, and PC, and prints the expected profit and cost to the government. Also the high and low profits and cost to the government are printed. The computer program uses Gaussian Quadrature as a numerical integration technique for calculation of the expected profit.



Fixed Price Incentive Firm, Cost-Profit Graph
Figure VI

APPENDIX IV

WORM1

08:16PDT

07/08/80

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100 DIMENSION AW(12),T(12),X1(12),X2(12),X3(12),F1(12),F2(12),F3(12)
110 DATA AW/.249147,.233492,.203167,.160078,.106939,.047175,
120&.047175,.106939,.160078,.203167,.233492,.249147/
130 DATA T/.125333,.367831,.587317,.769902,.904117,.98156,
140&-.98156,-.904117,-.769902,-.587317,-.367831,-.125333/
150 REAL MU
160 C(A,B,T)=(A+B)/2.+(B-A)/2.*T
170 F(A,B,MU,SIG,X)=(B-A)/2./SQRT(6.286)/SIG*EXP(-(X-MU)**2/(2.*SIG**2))
180 N=12
190 81 PRINT,'INPUT E(TC), VAR(TC),TARGET COST,TARGET PROFIT/
200 PRINT , ' ENTER ZEROES TO STOP/
210 INPUT ,MU,SIG,TC,TP
220 IF(MU.EQ.0)STOP
230 80 PRINT,'INPUT CONTRACTORS SHARE OF UNDER RUN, CONTRACTORS SHARE/
240 PRINT , 'OF OVER RUN, AND PRICE CEILING/
250 PRINT , 'ENTER ZEROES TO STOP/
270 INPUT, A,B,PC
280 IF(PC.EQ.0)GO TO 81
290 CC=(PC-TP-B*TC)/(1.-B)
300 IF(CC.GT.PC)PRINT,/CONTRACTOR MAY PAY COST OVER CEILING PRICE/
310 A1=MU-3.*SIG
320 B1=TC
330 A2=TC
340 IF(CC.GT.MU+3.*SIG)CC=MU+3.*SIG
350 B2=CC
360 A3=CC
370 B3=MU+3.*SIG
380 EPROF=0
390 DO1I=1,N
400 X1(I)=C(A1,B1,T(I))
410 X2(I)=C(A2,B2,T(I))
420 X3(I)=C(A3,B3,T(I))
430 F1(I)=F(A1,B1,MU,SIG,X1(I))
440 F2(I)=F(A2,B2,MU,SIG,X2(I))
450 1 F3(I)=F(A3,B3,MU,SIG,X3(I))
460 DO2I=1,N
470 EPROF=EPROF+((TP+A*(TC-X1(I)))*F1(I)+(TP-B*(X2(I)-TC))*F2(I))*AW(I)
480 IF(CC.GE.MU+3*SIG )GO TO 2
490 EPROF=EPROF+(PC-X3(I))*AW(I)*F3(I)
500 2 CONTINUE
510 EPRICE=EPROF+MU
520 IF(MU+3*S IG.GE.CC.AND.CC.LT.PC)GOTO4
530 PHIGH=TP-B*((MU+3*SIG)-TC)
540 GO TO 5
550 4 PHIGH=(PC-MU-3.*SIG)
560 5 PLOW=TP+A*(TC-MU+3.*SIG)
570 PRHIGH=MU+3.*SIG+PHIGH
580 PLOW=MU-3.*SIG+PLOW
590 PRINT 6,EPROF,EPRICE
600 PRINT 7,B3,PHIGH,PRHIGH
610 PRINT 8,A1,PLow,PRLOW
620 6 FORMAT(1X,'EXPECTED PROFIT ',E12.5,' EXPECTED PRICE ',E12.5)
630 7 FORMAT(1X,'HIGH COST ',E12.5,' PROFIT ',E12.5,' PRICE ',E12.5)
640 8 FORMAT(1X,' LOW COST ',E12.5,' PROFIT ',E12.5,' PRICE ',E12.5)
650 GO TO 80
660 END

```

READY

APPENDIX V

OLD WORM1

READY
RUN-20

WORM1 08:19PDT 07/08/80

INPUT E(TC), VAR(TC), TARGET COST, TARGET PROFIT
ENTER ZEROES TO STOP? 38.0266 .2877 38.5 3.85

INPUT CONTRACTORS SHARE OF UNDER RUN, CONTRACTORS SHARE
OF OVER RUN, AND PRICE CEILING
ENTER ZEROES TO STOP? .4 .4 46

EXPECTED PROFIT 0.40275E+01 EXPECTED PRICE 0.42054E+02
HIGH COST 0.38890E+02 PROFIT 0.71103E+01 PRICE 0.46000E+02
LOW COST 0.37163E+02 PROFIT 0.43846E+01 PRICE 0.41548E+02
INPUT CONTRACTORS SHARE OF UNDER RUN, CONTRACTORS SHARE
OF OVER RUN, AND PRICE CEILING
ENTER ZEROES TO STOP? .3 .3 47

EXPECTED PROFIT 0.39803E+01 EXPECTED PRICE 0.42007E+02
HIGH COST 0.38890E+02 PROFIT 0.81103E+01 PRICE 0.47000E+02
LOW COST 0.37163E+02 PROFIT 0.42510E+01 PRICE 0.41414E+02
INPUT CONTRACTORS SHARE OF UNDER RUN, CONTRACTORS SHARE
OF OVER RUN, AND PRICE CEILING
ENTER ZEROES TO STOP? 0 0 0 0

INPUT E(TC), VAR(TC), TARGET COST, TARGET PROFIT
ENTER ZEROES TO STOP? 0 0 0 0

PROGRAM STOP AT 220

USED 4.60 UNITS

APPENDIX VI

WORM3

09:17PDT

07/30/80

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100 DIMENSION A(8,5),ETC(5)
110 REAL MOR, MWR, MGOR
120 PRINT, 'THIS PROGRAM WAS WRITTEN TO PERFORM THE NECESSARY
130 PRINT, 'CALCULATIONS FOR A RISK ANALYSIS BY GEORGE WORM, 1980
140 PRINT, 'THE LINES REQUIRING THREE INPUTS END WITH L, ML, H
150 PRINT, 'MATERIAL COST L, ML, H
160 INPUT, A(1,1), A(1,2), A(1,3)
170 PRINT, 'MATERIAL OVERHEAD INDEPENDENT L, ML, H
180 INPUT, A(2,1), A(2,2), A(2,3)
190 PRINT, 'MATERIAL OVERHEAD RATE%
200 INPUT, MOR
210 MOR=MOR/100.
220 PRINT, 'INTERDIV TRSFERS L, ML, H
230 INPUT, A(3,1), A(3,2), A(3,3)
240 PRINT, 'DIRECT ENGRG LABOR (HOURS OR COST) L,ML,H
250 INPUT, A(4,1), A(4,2), A(4,3)
260 PRINT, 'ENGRG WAGE RATE (ENTER ONE IF LABOR IS COST AND NOT HOURS)
270 INPUT, EWR
280 PRINT, 'ENGRG OVERHEAD INDEPENDENT L, ML, H
290 INPUT, A(5,1), A(5,2), A(5,3)
300 PRINT, 'ENGRG OVERHEAD RATE%
310 INPUT, EOR
320 EOR=EOR/100.
330 PRINT, 'DIRECT MFG LABOR (HOURS OR COST) L, ML, H
340 INPUT, A(6,1), A(6,2), A(6,3)
350 PRINT, 'MGT WAGE RATE (ENTER ONE IF LABOR IN COST AND NOT HOURS)
360 INPUT, MWR
370 PRINT, 'MFG OVERHEAD INDEPENDENT L, ML, H
380 INPUT, A(7,1), A(7,2), A(7,3)
390 PRINT, 'MFG OVERHEAD RATE%
400 INPUT, MGOR
410 MGOR=MGOR/100.
420 PRINT, 'OTHER COSTS L, ML, H
430 INPUT, A(8,1), A(8,2), A(8,3)
440 PRINT, 'G AND A EXPENSE (PERCENT OF SUBTOTAL)%
450 INPUT, GAE
460 GAE=GAE/100.
470 DO I = 1,8
480 A(I,4) = (A(I,1) + 4. * A(I,2) + A(I,3))/6
490 1 A(I,5) = ((A(I,3) - A(I,1))/6.) **2
500 DO 2 I=1,4
510 ETC(I)=(1.+MOR)*A(1,I)+A(2,I)+A(3,I)+EWR*(1.+EOR)*A(4,I)+A(5,I)
520 ETC(I)=ETC(I)+MWR*(1.+MGOR)*A(6,I)+A(7,I)+A(8,I)
530 2 ETC(I)=ETC(I)*(1.+GAE)
540 ETC(5)=(1.+MOR)**2*A(1,5)+A(2,5)+A(3,5)+(EWR+EWR*EOR)**2*A(4,5)
550 ETC(5)=ETC(5)+A(5,5)+(MWR+MWR*MGOR)**2*A(6,5)+A(7,5)+A(8,5)
560 TSL = ETC(4) + 3. * SQRT (ETC(5))
570 RATIO = 3. * SQRT (ETC(5))/ETC(4)
580 IF (RATIO .GT. .05) GO TO 3
590 PRINT, 'SINCE VARIABILITY IS SMALL FFP IS RECOMMENDED
600 TP = 0
610 CR = 0
620 GO TO 4
630 3 PRINT, 'SINCE VARIABILITY IS MORE THAN 5 PERCENT FPIF IS RECOMMENDED

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```

640 RATIO=RATIO*100.
650 39 PRINT 5, ETC(4)
660 PRINT, 'INPUT WGM PROFIT (PERCENT)'
670 5 FORMAT (' EXPECTED TOTAL COST IS',F10.2)
680 INPUT, TP
690 TP=TP/100.
700 PRINT 6, TSL
710 6 FORMAT (' RISK ANALYSIS COST (UPPER LIMIT) IS',F10.2)
720 PRINT, 'INPUT COST RISK USED IN WGM (PERCENT)'
730 INPUT, CR
740 CR=CR/100.
750 4 PRINT 7
760 7 FORMAT (30X,'ESTIMATES FOR RISK ANALYSIS',///)
770 PRINT 8
780 8 FORMAT (2X, 'ELEMENTS', 41X, 'MOST')
790 PRINT 9
800 9 FORMAT (39X, 'MINIMUM', 4X, 'LIKELY', 3X, 'MAXIMUM')
810 PRINT 10, (A(1,I), I=1,3)
820 10 FORMAT (' MATERIAL',23X,'COST',3F10.2)
830 PRINT 11, (A(2,I), I=1,3)
840 11 FORMAT (' MGT OVERHEAD', 11X, 'INDEPENDENT', 3F10.2)
850 PRINT 12, MOR
860 12 FORMAT (2X, 'RATE FOR MATERIAL' F6.3)
870 PRINT 13, (A(3,I), I=1,3)
880 13 FORMAT (' INTERDIV TRSFERS', 16X, 'COST', 3F10.2)
890 PRINT 14, (A(4,I), I=1,3)
900 14 FORMAT (' DIRECT ENGRG LABOR', 12X, 'HOURS' 3F10.2)
910 PRINT 15, EWR
920 15 FORMAT (2X, 'WAGE RATE', F14.3)
930 PRINT 16, (A(5,I), I=1,3)
940 16 FORMAT (' ENGRG OVERHEAD', 10X, 'INDEPENDENT', 3F10.2)
950 PRINT 17, EOR
960 17 FORMAT(2X,'RATE FOR ENGRG',F9.3)
970 PRINT 18,(A(6,I),I=1,3)
980 18 FORMAT (' DIRECT MFG LABOR', 14X, 'HOURS', 3F10.2)
990 PRINT 19, MWR
1000 19 FORMAT (2X, 'WAGE RATE', F14.3)
1010 PRINT 20, (A(7,I), I=1,3)
1020 20 FORMAT (' MFG OVERHEAD', 12X, 'INDEPENDENT', 3F10.2)
1030 PRINT 21,MGOR
1040 21 FORMAT (2X, 'RATE FOR MFG', F11.3)
1050 PRINT 22, (A(8,I), I=1,3)
1060 22 FORMAT (' OTHER COST', 21X, 'COST', 3F10.2)
1070 PRINT 23,GAE
1080 23 FORMAT (' G&A EXPENSE', F14.4,////)
1090 PRINT 24
1100 24 FORMAT (10X, 'SUMMARY, CEILING/SHARE COMPUTATION')
1110 PRINT 25,ETC(1)
1120 25 FORMAT (/ ' SUMMARY, MINIMUM COST', 24X,F10.2)
1130 PRINT 26, ETC(2)
1140 26 FORMAT (/ ' SUMMARY, MOST LIKELY COST', 20X,F10.2)
1150 PRINT 27, ETC(3)
1160 27 FORMAT (/ ' SUMMARY, MAXIMUM COST', 24X,F10.2)
1170 PRINT 28,ETC(4)
1180 28 FORMAT (/ ' EXPECTED TOTAL COST, E(TC)',19X, F10.2,1X, 'EXCEEDS
W/PROB OF 50X')

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```

1190 PRINT 29, TSL
1200 29 FORMAT (/ / RISK ANALYSIS COST, RAC', 22X,F10.2,1X, 'EXCEEDED W/PROB
1210 IF(TP.EQ.0) GO TO 31                                OF 1% OR LESS')
1220 WP=TP-CR
1230 WPD=TSL*WP
1240 PRINT 30, WPD
1250 30 FORMAT (/ / WARRANTED PROFIT' 29X,F10.2)
1260 TPD=TP*ETC(4)
1270 PRINT 32, TPD
1280 32 FORMAT (/ / TARGET PROFIT', 32X,F10.2)
1290 CP=TSL*WPD
1300 PRINT 33, CP
1310 33 FORMAT (/ / CEILING PRICE', 32X,F10.2)
1320 PRINT 34, RATIO
1330 34 FORMAT (/ / PERCENT DIFFERENCE BETWEEN RAC AND OBJECTIVE', 1X,F10.2,
1340 PRINT 35                                             %')
1350 35 FORMAT (/ /, ' SHARING COMPUTATION:')
1360 DUMM=TPD-WPD
1370 PRINT 36, DUMM
1380 36 FORMAT (/ , 4X, 'WGM PROFIT LESS WARRANTED PROFIT', 9X,F10.2)
1390 DUM=TSL-ETC(4)
1400 PRINT 37, DUM
1410 37 FORMAT (/ , 4X, 'RISK ANALYSIS COST LESS OBJECTIVE COST', 3X,F10.2)
1420 CS=DUMM/DUM*100.
1430 PRINT 38, CS
1440 38 FORMAT (/ , 4X, 'CONTRACTORS SHARE' 24X,F10.2, '%')
1450 31 PRINT, 'TO CHANGE WGM PROFIT OR RISK, (TYPE 0 FOR YES, 1 FOR NO)'
1460 INPUT, ANS
1470 IF (ANS.EQ.0) GO TO 39
1480 STOP
1490 END

```

APPENDIX VII

OLD WORM3

READY
RUN-20

WORM3 09:12 PDT 07/30/80

THIS PROGRAM WAS WRITTEN TO PERFORM THE NECESSARY
CALCULATIONS FOR A RISK ANALYSIS BY GEORGE WORM, 1980
THE LINES REQUIRING THREE INPUTS END WITH L, ML, H
MATERIAL COST L, ML, H?8400 9000 120 00

MATERIAL OVERHEAD INDEPENDENT L, ML, H?0 0 0

MATERIAL OVERHEAD RATE%?5

INTERDIV TRSFERS L, ML, H?1700 1800 230 0

DIRECT ENGRG LABOR (HOURS OR COST) L, ML, H?85 95 115

ENGRG WAGE RATE (ENTER ONE IF LABOR IS COST AND NOT HOURS)?11.5

ENGRG OVERHEAD INDEPENDENT L, ML, H?0 0 0

ENGRG OVERHEAD RATE%?70

DIRECT MFG LABOR (HOURS OR COST) L, ML, H?200 230 290

MGT WAGE RATE (ENTER ONE IF LABOR IN COST AND NOT HOURS)?11

MFG OVERHEAD INDEPENDENT L, ML, H?0 0 0

MFG OVERHEAD RATE%?170

OTHER COSTS L, ML, H?400 450 500

G AND A EXPENSE (PERCENT OF SUBTOTAL)%?10

SINCE VARIABILITY IS MORE THAN 5 PERCENT FPIF IS RECOMMENDED
EXPECTED TOTAL COST IS 23161.60
INPUT MGM PROFIT (PERCENT)?12

RISK ANALYSIS COST (UPPER LIMIT) IS 25514.65
 INPUT COST RISK USED IN WGM (PERCENT) 75

ESTIMATES FOR RISK ANALYSIS

ELEMENTS		MINIMUM	MOST LIKELY	MAXIMUM
MATERIAL	COST	8400.00	9000.00	12000.00
MGT OVERHEAD	INDEPENDENT	0.00	0.00	0.00
RATE FOR MATERIAL	0.050			
INTERDIV TRSFERS	COST	1700.00	1800.00	2300.00
DIRECT ENGRG LABOR	HOURS	85.00	95.00	115.00
WAGE RATE	11.500			
ENGRG OVERHEAD	INDEPENDENT	0.00	0.00	0.00
RATE FOR ENGRG	0.700			
DIRECT MFG LABOR	HOURS	200.00	230.00	290.00
WAGE RATE	11.000			
MFG OVERHEAD	INDEPENDENT	0.00	0.00	0.00
RATE FOR MFG	1.700			
OTHER COST	COST	400.00	450.00	500.00
G&A EXPENSE	0.1000			

SUMMARY, CEILING/SHARE COMPUTATION

SUMMARY, MINIMUM COST	20373.93
SUMMARY, MOST LIKELY COST	22427.07
SUMMARY, MAXIMUM COST	28887.37
EXPECTED TOTAL COST, E(TC)	23161.60 EXCEEDED
	W/PROB OF 50%
RISK ANALYSIS COST, RAC	25514.65 EXCEEDED
	W/PROB OF 1% OR LESS
WARRANTED PROFIT	1786.03
TARGET PROFIT	2779.39
CEILING PRICE	27300.67
PERCENT DIFFERENCE BETWEEN RAC AND OBJECTIVE	10.16%

SHARING COMPUTATION:

WGM PROFIT LESS WARRANTED PROFIT	993.37
RISK ANALYSIS COST LESS OBJECTIVE COST	2353.05
CONTRACTORS SHARE	42.22%

ESTIMATES FOR RISK ANALYSIS

ASD/PMF CASE NO. _____

DATE _____

ESTIMATES

	MINIMUM (L)	MOST LIKELY (ML)	MAXIMUM (H)	EXPECTED VALUE $\frac{L + 4ML + H}{6}$	CALCULATED VARIANCE $\left(\frac{H - L}{6}\right)^2$
--	-------------	------------------	-------------	---	---

SUBCOMPONENTS

MATERIAL

MATERIAL OVERHEAD

RATE FOR MATERIAL

☐ P1

COST				E(V1)	Var(V1)
INDEPENDENT				E(V2)	Var(V2)

INTERDIV TNSFRS

DIRECT ENGRG LABOR

WAGE RATE

☐ R1

COST				E(V3)	Var(V3)
HOURS				E(V4)	Var(V4)

ENGRG OVERHEAD

RATE FOR ENGRG

☐ P2

INDEPENDENT				E(V5)	Var(V5)
-------------	--	--	--	-------	---------

DIRECT MFG LABOR

WAGE RATE

☐ R2

HOURS				E(V6)	Var(V6)
-------	--	--	--	-------	---------

MFG OVERHEAD

RATE FOR MFG

☐ P3

INDEPENDENT				E(V7)	Var(V7)
-------------	--	--	--	-------	---------

OTHER COSTS

COST

				E(V8)	Var(V8)
--	--	--	--	-------	---------

SA EXPENSES

PERCENT OF SUBTOTAL

☐ P4

FORM I

RISK ANALYSIS WORKSHEET (EXPECTED VALUE)

	(COL 1)	(COL 2)	(COL 3) = (COL 1) * (COL 2)
(1 + P1)	1	E(V1)	
	1	E(V2)	
	1	E(V3)	
(R1 + R1*P2)	1	E(V4)	
	1	E(V5)	
(R2 + R2*P3)	1	E(V6)	
	1	E(V7)	
	1	E(V8)	
		TOTAL (COL 3)	
		(1 + P4)	
		EXPECTED TOTAL COST = E(TC) = (1 + P4) * TOTAL (COL 3)	

FORM II

RISK ANALYSIS WORKSHEET (VARIANCE)

(COL 1)	(COL 2)	(COL 3) = (COL 1) * (COL 2)
$(1 + P1)^2$	Var(V1)	
1	Var(V2)	
1	Var(V3)	
$(R1 + R1*P2)^2$	Var(V4)	
1	Var(V5)	
$(R2 + R2*P3)^2$	Var(V6)	
1	Var(V7)	
1	Var(V8)	
	TOTAL (COL 3)	
	$(1 + P4)^2$	
VARIANCE OF TOTAL COST = Var(TC) = $(1 + P4)^2$ * TOTAL (COL 3)		

FORM III